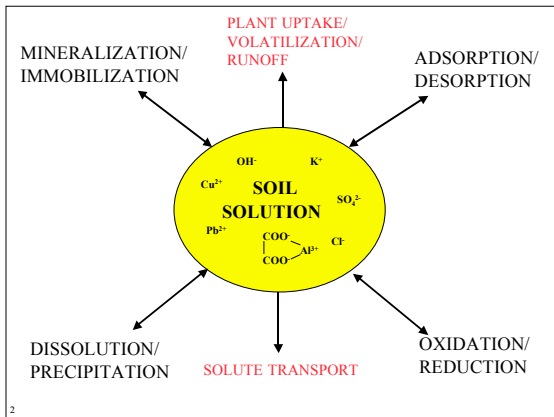


# ENSC 4734 Topic IV Solid-solution Equilibria

## TOPIC IV ENSC 4734

### SOLID-SOLUTION EQUILIBRIA

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### Solid-Solution Equilibria

- All minerals are soluble to some degree
- Mineral Solubility Principles
  - Elucidate pedogenic processes
  - Predict solution concentrations of elements
  - Predict trace-element bearing phases in both natural and contaminated systems
  - Verify remediation processes

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Complexation/Chelation

- Inorganic/organic ligands are ubiquitous in soils
- Large fraction of soluble metals may be complexed
- Most analytical methods measure total concentrations
  - No difference between free and complexed
- Influence mobility/toxicity

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Complexation/Chelation

- Good estimates of solution chemistry can be obtained with the following info:
  - Concentration of all elements in soil solution influencing speciation of target element
  - Stability constants ( $K_s$ ) or formation constants ( $K_f$ ) for all possible soluble complex formation reactions

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Complexation/Chelation

- Step wise stability constants
  - $M + L = ML$        $K_1 = [ML]/[M][L]$
  - $ML + L = ML_2$      $K_2 = [ML_2]/[ML][L]$
  - $ML_{n-1} + L = ML_n$     $K_n = [ML_n]/ML_{n-1}[L]$
- Overall stability constant for  $ML_n$ 
  - $\beta_n = [ML_n]/[M][L]^n$
  - $\beta_n = K_1K_2K_3 \dots K_n$

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Complexation/Chelation

- Example: Cd<sup>2+</sup> complexation by Cl<sup>-</sup>

Cd <sup>2+</sup> + Cl <sup>-</sup> = CdCl <sup>+</sup>	K <sub>1</sub> = 10 <sup>1.98</sup>
Cd <sup>2+</sup> + 2Cl <sup>-</sup> = CdCl <sup>0</sup> <sub>2</sub>	K <sub>2</sub> = 10 <sup>2.68</sup>
Cd <sup>2+</sup> + 3Cl <sup>-</sup> = CdCl <sup>-</sup> <sub>3</sub>	K <sub>3</sub> = 10 <sup>2.40</sup>
Cd <sup>2+</sup> + 4Cl <sup>-</sup> = CdCl <sup>2-</sup> <sub>4</sub>	K <sub>4</sub> = 10 <sup>2.50</sup>

Overall or cumulative complexation reactions

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Complexation/Chelation

- Example: Cd<sup>2+</sup> complexation by Cl<sup>-</sup>

$$K_n = \frac{[CdCl_n^{(2-n)}]}{[Cd^{2+}][Cl^-]^n}$$

Total Cd =

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Complexation/Chelation

- Example: Cd<sup>2+</sup> complexation by Cl<sup>-</sup>

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# ENSC 4734 Topic IV Solid-solution Equilibria

## Complexation/Chelation

- Example: Speciation of Pb in pH neutral soil leachate (Ion association Model)
- Pertinent ligands are  $\text{OH}^-$ ,  $\text{Cl}^-$ ,  $\text{SO}_4^{2-}$ ,  $\text{CO}_3^{2-}$  and fulvic acid (Ful $^-$ )
- Assume only a single ligand complexed to Pb:  $\text{PbCl}^+$
- $[\text{PbCl}^+] = K_{\text{rc1}}[\text{Pb}^{2+}][\text{Cl}^-]$

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## Complexation/Chelation

- Mass Balance expression for total Pb
- $\text{Pb}_T =$

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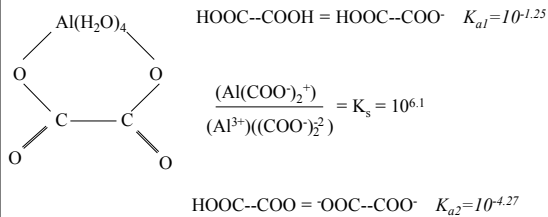
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## Complexation/Chelation

- Example: oxalic acid-Al hydroxide system



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Complexation/Chelation

- Extent of metal complexation is easily calculated
  - pH
  - ligand concentration
  - $Al^{3+}$  concentration

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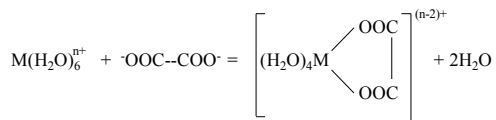
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Complexation/Chelation

- Chelates are favored thermodynamically



*CH<sub>3</sub>COO<sup>-</sup> only bonds weakly to Al<sup>3+</sup>, Why?*

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Complexation/Chelation

- Previous calculations are laborious
- More difficult in soil solutions (multiple ligands and metals)
- Simplified using computer speciation models
  - GEOCHEM-PC (Parker et al., 1994)
  - MINTEQA2 (Allison et al., 1991)
  - SOILCHEM (Sposito and Coves, 1988)

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Solubility Relationships

- K<sub>sp</sub> allows one to predict chemical composition of soil solution in equilibrium with particular mineral phase
- Example Cu<sup>2+</sup> solubility controlled by malachite dissolution
- $\text{Cu}_2(\text{OH})_2\text{CO}_3(\text{s}) + 2\text{H}^+ \rightarrow 2\text{Cu}^{2+} + \text{CO}_3^{2-} + 2\text{H}_2\text{O}$

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Solubility Relationships

- Equilibrium Expression
- $$K_{\text{diss}} = \frac{(\text{Cu}^{2+})^2(\text{CO}_3^{2-})(\text{H}_2\text{O})^2}{(\text{Cu}_2(\text{OH})_2\text{CO}_3)(\text{H}^+)^2}$$
- Equilibrium constant for malachite dissolution can be determined from standard Gibbs free energy of formation ( $\Delta G_f^\circ$ )

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Solubility Relationships

- $\Delta G_r^\circ = m\Delta G_f^\circ \text{products} - n\Delta G_f^\circ \text{reactants}$
- $\Delta G_r^\circ = 2\Delta G_f^\circ \text{Cu} + \Delta G_f^\circ \text{CO}_3 + 2\Delta G_f^\circ \text{H}_2\text{O} - \Delta G_f^\circ \text{mal}$
- $\Delta G_r^\circ = 2(65.52) + (-527.9) + 2(-237.114) - (-903.7) = 32.56 \text{ kJ mol}^{-1}$

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# ENSC 4734 Topic IV Solid-solution Equilibria

## Solubility Relationships

- $-RT \ln K = \Delta G_r^\circ$
- At 298.15 K (25°C) and  $R=8.314 \text{ J K}^{-1} \text{ mol}^{-1}$
- $K = 10^{-5.70}$

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## Solubility Relationships

- Next step is to determine  $\text{CO}_3^{2-}$  and proton concentration
- Proton measured with pH meter, 7.5
- Carbonate:  
 $-\text{CO}_2(\text{g}) + \text{H}_2\text{O} \rightarrow \text{CO}_3^{2-} + 2\text{H}^+$
- $K = 10^{-18.17} = \frac{(\text{CO}_3^{2-})(\text{H}^+)^2}{P_{\text{CO}_2}}$

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## Solubility Relationships

- $\text{Log}(\text{CO}_3^{2-}) = -18.17 + \text{log } P_{\text{CO}_2} + 2\text{pH}$
- Use  $P_{\text{CO}_2} = 10^{-2.5}$
- $\text{Log}(\text{CO}_3^{2-}) = -18.17 + -2.5 + 2(7.5) = -5.67$

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Solubility Relationships

- $(\text{Cu}^{2+})^2 = \frac{10^{-5.70}(\text{H}^+)^2}{\text{CO}_3^{2-}}$
- $(\text{Cu}^{2+}) = \frac{10^{-2.85}(\text{H}^+)}{(\text{CO}_3^{2-})^{0.5}}$
- $\text{Log}(\text{Cu}^{2+}) = -2.85 - 1/2\text{log}(\text{CO}_3^{2-}) - \text{pH}$
- $\text{Log}(\text{Cu}^{2+}) = -2.85 - 1/2(-5.67) - 7.5 = -7.52$

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Solubility Relationships

- Ion Activity Product and Relative Saturation
- Previous example determine activity of a component based on controlling solid phase
- Procedure can work in reverse
- Difficult to determine spectroscopically

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Solubility Relationships

- Ksp describes activity ratio of products to reactants that will exist in soln at equilibrium with a mineral
- Ksp is mineral specific
- Solution values called the IAP
- $\text{IAP} = \text{Ksp}$  if a specific mineral present and in equilibrium with soil components

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Solubility Relationships

- IAP is a property of the soil solution
  - Very difficult to accurately determine in soils
- Detail characterization of chemical composition of soil solution
  - Ion association model
  - Commercially available computer codes

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Solubility Relationships

- Example: lead contaminated soil
- SEM indicates lead associated with P
- Potentially 3 lead bearing phosphates
  - Lead phosphate ( $Pb(PO_4)_{0.67}$ )
  - Hydroxypyromorphite ( $Pb(PO_4)_{0.6}(OH)_{0.2}$ )
  - Plumbogummite ( $PbAl_3(PO_4)_2(OH)_5$ )

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Solubility Relationships

- Lead Phosphate
- $Pb(PO_4)_{0.67} \rightarrow Pb^{2+} + 0.67PO_4^{3-}$
- $K_{sp} = (Pb^{2+})(PO_4^{3-}) = 10^{-14.79}$

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Solubility Relationships

- Hydroxypyromorphite
- $\text{Pb}(\text{PO}_4)_{0.6}(\text{OH})_{0.2} + 0.2\text{H}^+ \rightarrow \text{Pb}^{2+} + 0.6\text{PO}_4^{3-} + 0.2\text{H}_2\text{O}$
- $$K_{\text{sp}} = \frac{(\text{Pb}^{2+})(\text{PO}_4^{3-})}{(\text{H}^+)^{0.2}} = 10^{-12.56}$$

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Solubility Relationships

- Plumbogummite
- $\text{PbAl}_3(\text{PO}_4)(\text{OH})_5 \cdot \text{H}_2\text{O} + 5\text{H}^+ \rightarrow \text{Pb}^{2+} + 3\text{Al}^{3+} + 2\text{PO}_4^{3-} + 6\text{H}_2\text{O}$
- $$K_{\text{sp}} = \frac{(\text{Pb}^{2+})(\text{Al}^{3+})^3(\text{PO}_4^{3-})^2}{(\text{H}^+)^5} = 10^{-32.79}$$

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Solubility Relationships

- Detailed chemical analysis and GEOCHEM-PC yielded:
  - $[\text{Pb}^{2+}] = 10^{-7.25}$
  - $[\text{PO}_4^{3-}] = 10^{-13.13}$
  - $[\text{Al}^{3+}] = 10^{-9.86}$
  - pH = 5.97

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Solubility Relationships

- IAP compared to Ksp
- Relative saturation:  $\Omega = \frac{IAP}{Ksp}$
- Saturation index =  $\log IAP - \log Ksp$

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Solubility Relationships

- IAP is between Ksp and 0.1 Ksp ( $0.1 < \Omega < 1$  and  $-1 < SI < 0$ ) saturated
- $IAP < 0.1Ksp$  ( $\Omega < 0.1$  and  $SI < -1$ ) undersaturated
- $IAP > Ksp$  ( $\Omega > 1$  and  $SI > 0$ ) supersaturated

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Solubility Relationships

- Plumbogummite present assuming the following:
  - Equilibrium exists
  - Mineral activities unity
  - Soil water activity is unity
  - STP
  - All significant components in soil solution determined and modeled

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Solubility Relationships

- Deviations from solubility relationships
  - Metastability and the step rule
  - Poorly and microcrystalline solids
  - Solid solutions

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Stability diagrams

- Used to determine which solid phase(s) control the conc. of an element in the soil solution
- Constructed by converting solubility relationships into log terms and rearranging to form straight lines
- Advantage - several different solid phases can be compared at one time
  - Comparing IAP to  $K_{sp}$  for each possible mineral phase extremely tedious

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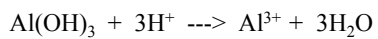
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Stability diagrams

- Example: Gibbsite Dissolution

Calculate K from standard  $\Delta G$  values



$$\Delta G_{\text{rxn}} = [(-491.61) + 3(-237.52)] - [(1158.24) + 3(0)]$$
$$= -45.92 \text{ kJ mole}^{-1}$$

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# ENSC 4734 Topic IV Solid-solution Equilibria

## Stability diagrams

- Example: Gibbsite Dissolution

$$\Delta G_{\text{rxn}} = -RT \ln K =$$

$$-(0.00831 \text{ kJ K}^{-1} \text{ mole}^{-1})(298\text{K})(2.303) \log(K)$$

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## Stability diagrams

- $\text{Al(OH)}_3_{\text{gibbsite}} + 3\text{H}^+ = \text{Al}^{3+} + 3 \text{H}_2\text{O}$
- $(\text{Al}^{3+})/(\text{H}^+)^3 = 10^{8.04}$   
or
- $\log \text{Al}^{3+} = 8.04 - 3\text{pH}$

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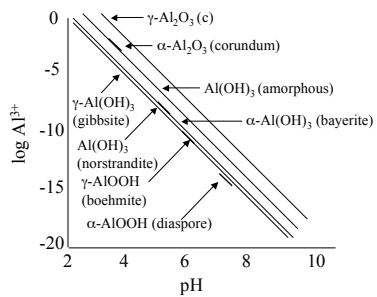
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## Stability diagrams



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Stability diagrams

- Line nearest the axis the most insoluble
- Solubility Al-oxides > corundum > bayerite > boehmite > nost randite > gibbsite > diaspore
- Activity of Al<sup>3+</sup> very pH dependent
  - decrease 100 fold for each unit increase in pH

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Stability diagrams

- Mineralogical characterization of sewage sludge: Pb associated with either silica or phosphate
- Two plausible mineral phases
  - Hydroxypyromorphite [Pb<sub>5</sub>(PO<sub>4</sub>)<sub>3</sub>OH]
  - Alamosite [PbSiO<sub>3</sub>]

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Stability diagrams

- Hydroxypyromorphite
  - $\text{Pb}_5(\text{PO}_4)_3(\text{OH})_{(s)} + 7 \text{H}^+ \rightarrow 5\text{Pb}^{2+} + 3\text{H}_2\text{PO}_4^- + \text{H}_2\text{O}$
- Alamosite
  - $\text{PbSiO}_3(s) + 2\text{H}^+ + \text{H}_2\text{O} \rightarrow \text{Pb}^{2+} + \text{H}_4\text{SiO}_4$

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Stability diagrams

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Stability diagrams

- Aluminosilicate minerals
- Can be expressed in terms of  $Al^{3+}$ ,  $H^+$ ,  $H_4SiO_4$  (e.g. kaolinite)

$Al_2Si_2O_5(OH)_4 + 6H^+ = 2Al^{3+} + 2H_4SiO_4 + H_2O$   
 Equilibrium Constant:  $K = 10^{5.45}$

$$\frac{(Al^{3+})_2(H_4SiO_4)_2}{(H^+)^6} = 10^{5.45}$$

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Stability diagrams

- $2 \log Al^{3+} + 6 \text{pH} = 5.45 - 2 \log H_4SiO_4$
- $\log Al^{3+} + 3\text{pH} = 2.73 - \log H_4SiO_4$
- Plot  $\log Al^{3+} + 3 \text{pH}$  on y-axis and  $\log H_4SiO_4$  on x-axis
- Selecting diff values for  $\log H_4SiO_4$

Log $H_4SiO_4$	$\log Al^{3+} + 3\text{pH}$
-2.5	5.23
-4.0	6.73
-5.5	8.23

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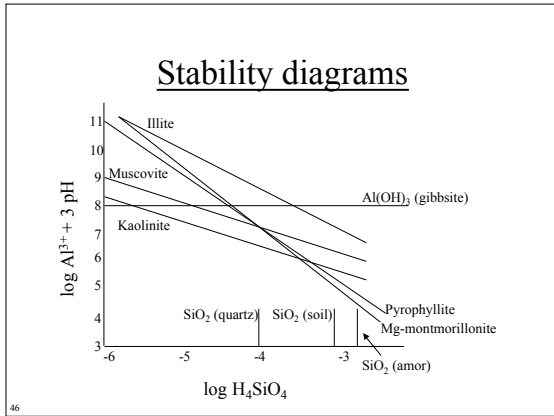
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# ENSC 4734 Topic IV Solid-solution Equilibria



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- ### Stability diagrams
- Soils are in a general state of dis-equilibrium
    - steady states only locally established
  - Dis-equilibrium due to
    - slow rates of mineral neoformation/alteration
    - incongruent and slow dissolution rates
    - open system (leaching)
  - Thermodynamics vs. kinetics
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